Geometric Structures in Integrable Systems

Sergey Adzhiev (with Victor Vedenyapin), Entropy for generalised quantum kinetic and Liouville equations.

Abstract: In his 1872 paper L. Boltzmann proved H-theorem for a class of kinetic equations. We generalize those equations in different manners in particular for quantum case. In 1906 H. Poincaré and in 2006 V. Kozlov and D. Treschev proposed new form of H-theorem for Liouville equation. We generalize their results in different directions. In particular we propose a theorem: time average coincides with Boltzmann extreme point. This theorem justifies (and corrects) the Gibbs method even in nonergodic case.

Alexander Aksenov (with A.A. Kozyrev), Reductions of stationary boundary layer equation.

Abstract: P. Clarkson and M. Kruskal (1989) proposed a method for finding reductions of partial differential equations with two independent variables. For the Boussinesq equation new similarity reductions were obtained. This reductions cannot be obtained using classical Lie group method. In our paper more simple method for finding reductions of partial differential equations with two independent variables is proposed. Examples of application this method are given. For stationary boundary layer equation new similarity reductions are obtained.

Michael Babich, Canonical parameterization of coadjoint orbits of $GL(N, C)$ with complicated Jordan structure and isomonodromic deformation equations.

Abstract: I will demonstrate that (co)adjoint orbit of $GL(N, C)$ has a structure of a symplectic fibration. It can be used for the construction of the rational Darboux coordinates on the orbit. Isomonodromic deformation equations are defined on the symplectic quotient of the product of such orbits. The iteration procedure for the solving of the momentum-level equation and the simultaneous factorization with respect to the diagonal $GL(N, C)$-action will be presented. The method works for a wide class of matrices. The isomonodromic deformations of the Fuchsian equation with the rank-one traceless matrix-residues (their Jordan forms consist of one 2×2 Jordan block with zero diagonal and $(N-2) \times (N-2)$-dimension zero block) will be considered as an example.

Yuri Brezhnev, How to papameterize algebraic dependencies?.

Abstract: We propose the unified and geometric formulation to the uniformization theory of Riemann surfaces and orbifolds of finite genera. Complete description is based on the standard Fuchsian differential equations but is extended to the Abelian integrals and analytic connection on a cotangent bundle over the surfaces/orbifolds. The invariant (geometric) description reduces to a fundamental system of differential equations for a set of holomorphic integrals. We exhibit the first explicitly solvable example: the case when holomorphic integrals on a Riemann surface of genus $g = 2$, as functions of the uniformizing variable, is analytically representable in terms of known functions.

Elena Bunkova (with V.M. Buchstaber), Introducing a new notion of algebraical integrability.

Abstract: Let us consider the general homogeneous quadratic dynamical system. We will call it algebraically integrable by given functions $h_1, \dots, h_n$ if the set of roots of the equation $\xi^n - h_1\xi^{n-1} + \dots + (-1)^n h_n \equiv 0$ solves the dynamical system.

The talk introduces this new notion of algebraic integrability and presents a wide class of quadratic dynamical systems that are algebraically integrable by the set of functions $h_1, \dots, h_n$ where h_1 is the solution to an ordinary differential equation of order n and h_k are differential polynomials in h_1, $k = 2, \dots, n$. Results on algebraically integrable quadratic dynamical systems and non-linear ordinary differential equations related to them are obtained. Classical examples like the Darboux-Halphen system are considered.

Gennady El, Kinetic equation for a soliton gas – a new integrable system?

Abstract: In 1971 V.E. Zakharov introduced a kinetic equation describing dynamics of a spectral distribution function in a rarefied soliton gas – an infinite random ensemble of KdV solitons distributed on the line with nonzero small density [1]. The finite-density generalization [2] of Zakharov's equation represents a nonlinear integro-differential equation, which was shown in [3] to be related to the infinite-genus, thermodynamic limit of the Whitham modulation systems associated with finite-gap solutions of the KdV equation. Recent studies [4], [5] have revealed a number of remarkable properties of the new kinetic equation, which, in particular, has been shown to possess an infinite number of integrable hydrodynamic reductions. This is a strong evidence in favour of integrability of the full kinetic equation in the sense yet to be understood. Construction of kinetic theory of soliton gases is part of the general programme of the development of turbulence theory in integrable systems [6].

References:

[1] Zakharov, V.E., Kinetic equation for solitons. Sov. Phys. JETP 33, 538–541 (1971) [2] El, G.A., Kamchatnov, A.M., Kinetic equation for a dense soliton gas, Phys. Rev. Lett. 95, 204101 (2005) [3] El, G.A., The thermodynamic limit of the Whitham equations, Phys. Lett. A, 311, 374-383 (2003) [4] El, G.A., Kamchatnov, A.M., Pavlov, M.V., and Zykov, S.A., Kinetic equation for a soliton gas and its hydrodynamic reductions, J. Nonlin. Sci 21 151-191, (2011). [5] El, G.A., Pavlov, M.V. and Taranov, V.B., Generalised hydrodynamic reductions of the kinetic equation for soliton gas, Theor. Math. Phys. 171 675–682 (2012).

[6] Zakharov, V.E., Turbulence in integrable systems. Stud. Appl. Math. 122, 219–234 (2009)

Maria Elaeva (with B.A. Dubrovin), Asymptotic and numerical methods in study of the generalized Burgers equation.

Abstract: We consider the generalized Burgers equation depending on a small parameter $\varepsilon > 0$. For this equation we obtain asymptotic formula and present some heuristic arguments justifying its validity. Also we give numerical evidences supporting our conjectures. The finite element method was used for solving this equation. The finite element solution is compared with the asymptotic formula.

Georgi Grahovski, On N-wave equations and their gauge equivalent.

Abstract: The class of nonlinear evolution equations (NLEE) - gauge equivalent to the N-wave equations related to the simple Lie algebra g are derived and analyzed. The corresponding Lax pairs and the time evolution of the scattering data are found. The Zakharov-Shabat dressing method is appropriately modified to construct their soliton solutions. Several examples including ones describing isoparametric hypersurfaces are presented. The hierarchy of the Hamiltonian structures to the gauge equivalent systems to the N-wave ones is also discussed.

Peter Grinevich (with P.M. Santini), Holomorphic eigenfunctions of the vector field associated with the dispersionless Kadomtsev-Petviashvili equation.

Abstract: Vector fields naturally arise in many branches of mathematics and physics. Recently it was discovered that Lax pairs for many important multidimensional integrable partial differential equations (PDEs) of hydrodynamic type (also known as dispersionless PDEs) consist of vector field equations. These vector fields have complex coefficients and their analytic, in the spectral parameter, eigenfunctions play an important role in the formulations of the direct and inverse spectral transforms. In this paper we prove existence of eigenfunctions of the basic vector field associated with the celebrated dispersionless Kadomtsev-Petviashvili equation, which are holomorphic in the spectral parameter λ in the strips $|\Im\lambda| > C_0$.

Anatoly Kamchatnov, Oblique solitons.

Abstract: Oblique solitons are two-dimensional dark solitons generated, for example, by a flow of Bose-Einstein condensate past an obstacle. They were predicted in [1] and observed in experiments with cavity-polariton condensates in [2]. For many years the common belief was that two-dimensional dark solitons are unstable with respect to so-called "snake" instability and this belief was confirmed by the theory and experiment for the case of dark solitons propagating in quiescent medium. Thus, seeming contradiction appeared and the talk is devoted to resolution of this "contradiction" in framework of the theory of convective instability of dark solitons developed in [3,4].

References:
[1] G. A. El, A. Gammal, A. M. Kamchatnov, Oblique dark solitons in supersonic flow of a Bose-Einstein condensate, Phys. Rev. Lett. 97, 180405 (2006). [2] A. Amo et al., Polariton superfluids reveal quantum hydrodynamic solitons, Science **332**, 1167 (2011). [3] A. M. Kamchatnov and L. P. Pitaevskii, Stabilization of solitons generated by a supersonic flow of Bose-Einstein condensate past an obstacle, Phys. Rev. Lett. 100, 160402 (2008). [4] A. M. Kamchatnov and S. V. Korneev, Condition for convective instability of dark solitons, Phys. Lett. A 375, 2577 (2011).

Oleg Kaptsov (with A.V. Schmidt), Non-invariant solutions of the three-dimensional semi-empirical model of the far turbulent wake.

Abstract: A semi-empirical three-dimensional model of turbulence in the approximation of the far turbulent wake behind a self-propelled body in a passive stratified medium is considered. The sought quantities are the kinetic turbulent energy, kinetic energy dissipation rate, averaged density defect and density fluctuation variance. The full group of transformations admitted by this model is found. The model is reduced to a system of ordinary differential equations using symmetry groups and B-determining equations method. This system of ordinary differential equations satisfying natural boundary conditions was solved numerically. The obtained solutions agree with experimental data.

Rinat Kashaev, Faddeev's quantum dilogarithm and 3-manifold invariants.

Abstract: Faddeev's quantum dilogarithm is a special function satisfying a remarkable five-term quantum identity. It finds applications in quantum integrable systems and quantum topology. I will outline its role in constructing combinatorial TQFT's of novel type and describe connections to topology and geometry of hyperbolic 3-manifolds.

Olga Kravchenko, Knots and clusters.

Abstract: In 2000 Fomin and Zelevinsky introduced a new formalism in representation theory - the cluster algebras. Its beauty lies in its relations to many different subjects, in particular to Poisson geometry and discrete integrable systems. I will discuss cluster variables and Poisson brackets related to knots.

Michal Marvan, Minimal sets of compatibility conditions of orthonomic systems.

Abstract: Given an overdetermined system of PDE solved for leading derivatives, we provide an efficient geometric description of redundancy-free sets of its compatibility conditions.

Alexander Mikhailov, Darboux transformations and symmetries of partial difference equations.

Abstract: We study Darboux transformations corresponding to Lax operators on automorphic sl_2 Lie algebras. We present the integrable differential difference and partial difference equations associated with these Darboux transformations. Starting from a Darboux transformation we derive a recursion operator for the hierarchy of symmetries for the corresponding differential difference and partial difference equations.

Andrey Mironov, Self-adjoint commuting ordinary differential operators of rank two.

Abstract: Self-adjoint commuting ordinary differential operators of rank two are considered. We find sufficient conditions when an operator of fourth order commuting with an operator of order $4g + 2$ is self-adjoint. An equation on potentials $V(x), W(x)$ of the self-adjoint operator $L = (\partial_x^2 + V)^2 + W$ and some additional data is introduced. With the help of this equation examples are constructed.

Paulius Miskinis, Weakly nonlocal supersymmetric KdV hierarchy.

Abstract: By using the fractional derivative in Caputo sense a weakly nonlocal generalization of superKdV, which possesses also an unlimited number of conservation laws and exact solutions are presented. The correlation to the equation with quadratic nonlinearity are shown.

Oleg Mokhov (with N.A. Pavlenko), On Hamiltonian geometry of the associativity equations.

Oleg Morozov, Lie Pseudo-Groups and Zero-Curvature Representations of Differential Equations.

Abstract: In this talk I will discuss the interplay between Cartan's structure theory of Lie pseudo-groups and zero-curvature representations of integrable (2+1) nonlinear differential equations.

Malik Negmatov (with Victor Vedenyapin), On Liouville and Vlasov equations, and its hydrodynamic consequences.

Abstract: We describe the derivation of Vlasov–Maxwell equation from classical Lagrangian, and a similar derivation of the Vlasov–Poisson–Poisson charged gravitating particles. We derive electromagnetic hydrodynamic equations and present them to the Godunov's double divergence form. For them we get generalized Lagrange identity and compare it. Analyzes the steady-state solutions of the Vlasov–Poisson–Poisson equation: their types is changing at a certain critical mass having a clear physical meaning. The consequence is the different behavior of particles - recession or collapse trajectories. We investigate topology of solutions of those hydrodynamic equations due to Arnold–Kozlov commuting vector fields, and connection to Hamilton–Jacobi method.

Dmitry Novikov, Theta-function solutions of the Belavin-Polyakov-Zamolodchikov equation and the Hamilton-Jacobi equation.

Abstract: The system of equations for elliptic solutions of the BPZ equation is considered. The Hamilton-Jacobi equation, that correspond to Painleve VI, includes in this system.

Alexei V. Penskoi, Extremal metrics for Laplace eigenvalues on tori.

Abstract: Given a closed compact surface, eigenvalues of the Laplace-Beltrami operator are functionals on the space of Riemannian metrics of fixed area on this surface.

The question about extremal metrics for these eigenvalues is a difficult problem representing an interesting interplay between minimal surfaces, Lie group symmetries and the classical differential equations.

In this talk we shall describe significant advances is this domain happened during last years.

Ziemowit Popowicz, Connections of the coupled two-component Harry Dym system with the coupled two-component KdV equation.

Abstract: Four different two-component coupled Harry Dym equations and its Lax representation is given. All these equations are transformed, via the reciprocal link, to the integrable coupled KdV equation. The first two-component Harry Dym equation is connected with the Hirota–Satsuma equation, the second with the Drinfeld–Sokolov equation, the third gives us new system of coupled KdV equations and the last reduces to the know symmetric system of coupled KdV equations.

Zoran Rakic (with Yury Nikolayevsky), On Osserman condition in pseudo-Riemannian geometry.

Abstract: Let (M, g) be a pseudo-Riemannian manifold, with curvature tensor R. The Jacobi operator R_X is the symmetric endomorphism of T_pM defined by $R_X(Y) = R(Y, X)X$. In Riemannian settings, if M is locally a rank-one symmetric space or if M is flat, then the local isometry group acts transitively on the unit sphere bundle SM and hence the eigenvalues of R_X are constant on SM. Osserman in the late eighties, wondered if the converse held; this question is usually known as the *Osserman conjecture*. In the last twenty years many authors have been studied problems which arising from the Osserman conjecture and its generalizations. In the first part of the lecture we will give an overview of Osserman type problems in the pseudo-Riemannian geometry. The second part is devoted to the equivalence of the Osserman pointwise condition and the duality principle.

Vladimir Salnikov (with Thomas Strobl), Gauged sigma models and graded geometry.

Abstract: We study some graded geometric constructions appearing naturally in the context of gauge theories. In terms of Q-bundles we describe the gauge transformations of the (twisted) Poisson and Dirac sigma models.

Inspired by a known relation of gauging with equivariant cohomology we generalize the latter notion to the case of arbitrary Q-manifolds. This permits to obtain the mentioned sigma by gauging essentially infinite dimensional groups and describe their symmetries in terms of classical differential geometry. This approach can also be useful to study supersymmetric gauge theories.

Vladimir Sokolov, Integrable nonhomogenious hydrodynamic type systems.

Abstract: 1+1 nonhomogenious hydrodynamic type systems equipped by a Lax representation with movable singularities are investigated. A wide class of examples for arbitrary number of independent variables is presented.

Iskander Taimanov (with S.P. Tsarev), Faddeev eigenfunctions for two-dimensional Schrodinger operators via the Moutard transformation.

Abstract: We demonstrate how the Moutard transformation of two-dimensional Schrodinger operators acts on the Faddeev eigenfunctions on the zero energy level and present some explicitly computed examples of such eigenfunctions for smooth fast decaying potentials of operators with non-trivial kernel and for deformed potentials which correspond to blowing up solutions of the Novikov-Veselov equation.