A COMBINATORIAL MODEL OF THE LIPSHITZ METRIC FOR SURFACES WITH PUNCTURE

VLADIMIR SHASTIN

In the work [1] Ivan Dynnikov described a polynomial algorithm for the solution of the word problem in mapping class groups of punctured surfaces, where as the size of the algorithm's input he uses a modified version of the word length function of the mapping class group. Namely for a finite generating set $\mathcal{A}$ of the mapping class group of a punctured surface S Dynnikov defined *the zipped word length function* $\mathrm{zwl}_{\mathcal{A}}$ as follows:

$$\mathrm{zwl}_{\mathcal{A}}(\varphi) = \min_{\substack{\varphi = a_1^{k_1} \dots a_m^{k_m} \\ a_1, \dots, a_m \in \mathcal{A} \\ k_1, \dots, k_m \in \mathbb{Z}}} \sum_{i=1}^{m} \log_2(|k_i| + 1),$$

where $\varphi \in \mathrm{MCG}(S)$.

For special generating sets $\mathcal{A}$ he proved that the word problem is efficiently solvable with respect to $\mathrm{zwl}_{\mathcal{A}}$:

Theorem (Dynnikov). *Let S be a compact surface, $\mathcal{P} = (P_1, \dots, P_n) \in S$ a non-empty collection of pairwise distinct points such that the mapping class group $G = MCG(S \setminus \mathcal{P})$ is infinite. Let $\mathcal{A}$ be a finite generating set for G such that*

1. *every element in $\mathcal{A}$ is a fractional power of a Dehn twist;*
2. *every Dehn twist from G is conjugate to a fractional power of an element from $\mathcal{A}$*

Then the word problem in G is solvable in polynomial time with respect to $zwl_{\mathcal{A}}$.

The function $\mathrm{zwl}_{\mathcal{A}}$ determines the right-invariant metric $\rho_{\mathcal{A}}$ on $\mathrm{MCG}(S)$ as follows:

$$\rho_{\mathcal{A}}(\varphi, \psi) = \mathrm{zwl}_{\mathcal{A}}(\psi \varphi^{-1}),$$

where $\varphi, \psi \in \mathrm{MCG}(S)$.

It turns out that this metric is closely related to the Lipshitz metric on the Teichmüller space. In this talk we describe this relation and give the proof of the following theorem:

Theorem. *Let S be an oriented surface with non-empty set of punctures, ϵ a positive constant, σ a hyperbolic structure on S, lying in the ϵ-thick part of the Teichmüller space $\mathcal{T}_\epsilon(S)$, and $\mathcal{A}$ a finite generating set of $MCG(S)$ with the following properties:*

1. *every element in $\mathcal{A}$ is a fractional power of a Dehn twist;*
2. *every Dehn twist from G is conjugate to a fractional power of an element from $\mathcal{A}$*

Let also $i_\sigma \colon MCG(S) \to \mathcal{T}_\epsilon(S)$ be the map that sends $\varphi \in MCG(S)$ to the image of σ under φ. Then i_σ is a quasi-isometry from $MCG(S)$ equipped with the metric $\rho_{\mathcal{A}}$ to the thick part of $\mathcal{T}(S)$ equipped with the Lipshitz metric.

Список литературы

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Department of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow 119991, Russia.
Laboratory of Quantum Topology, Chelyabinsk State University, Brat'ev Kashirinykh street 129, Chelyabinsk 454001, Russia.

E-mail address: vashast@gmail.com