ON CERTAIN TORIC SPACES ARISING FROM SIMPLE POLYTOPES

IVAN LIMONCHENKO

To any convex simple n-dimensional polytope P with m facets one can associate its moment-angle manifold $\mathcal{Z}_P$ – one of the main objects of study in toric topology. This manifold was introduced firstly by M.Davis and T.Januszkiewicz as a generalization of the notions of a quasitoric manifold and a projective toric manifold. V.Buchstaber and T.Panov proved that $\mathcal{Z}_P$ is a smooth $(m+n)$-dimensional closed 2-connected manifold with a compact torus T^m action, whose orbit space is homeomorphic to the polytope P itself, and cohomology algebra of $\mathcal{Z}_P$ is isomorphic to the Tor-algebra $Tor_{k[v_1,\ldots,v_m]}(k[P],k)$ of P over a polynomial algebra, when k is a field or the ring of integers. Studying Massey products in the Tor-algebras is related to a graded version of a classical J.-P.Serre problem on rationality of Poincaré series for Noetherian local rings being studied since 1960s. The topology of $\mathcal{Z}_P$ is governed by the face lattice of P and can be very complicated.

In our talk we shall introduce several equivalent definitions of $\mathcal{Z}_P$ arising in toric and symplectic geometry, their relation to smooth toric varieties, and then discuss higher Massey products in cohomology and rational formality of moment-angle manifolds $\mathcal{Z}_P$ when P is a 2-truncated cube, that is a consecutive cut of only codimension 2 faces starting with a cube. We introduce a family of n-dimensional 2-truncated cubes P, such that there is a nontrivial n-fold Massey product in cohomology of the moment-angle manifold $\mathcal{Z}_P$ for any $n \geqslant 2$. V.Buchstaber and V.Volodin proved that any flag nestohedron can be realized as a 2-truncated cube; we present our family of 2-truncated cubes as flag nestohedra for $n \geqslant 3$. Then we discuss some results and problems concerning nontrivial triple Massey products for $\mathcal{Z}_P$ when P is a graph-associahedron of M.Carr and S.Devadoss or a generalized associahedron of S.Fomin and A.Zelevinsky, based on the properties of these polytopes arising in representation theory, cluster algebras and convex geometry.

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Department of Geometry and Topology, Faculty of Mechanics and Mathematics, Moscow State University, Leninskiye Gory, Moscow, 119991, Russia
E-mail address: ilimonchenko@gmail.com